

Mathematical Analysis - List 16

1. Find the indicated partial derivative. Assume the variables are restricted to a domain on which the function is defined.

a) $\frac{\partial F}{\partial m_2}$ and $\frac{\partial F}{\partial r}$ if $F = \frac{Gm_1m_2}{r^2}$; b) $\frac{\partial m}{\partial v}$ if $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$;

c) z_x and z_y if $z = x^9 + 3^2y + x^y$; d) z_y if $z = \frac{4x^2y^7 - y^2}{15xy - 8y}$;

e) $\frac{\partial}{\partial \lambda} \left(\frac{x^2y\lambda - 3\lambda^5}{\sqrt{\lambda^2 - 3\lambda + 7}} \right)$; f) $\frac{\partial}{\partial w} \left(\frac{x^2yw - 3xy^3w^5}{3w^2 + 2} \right)^{-5/2}$.

2. Show that the function $Q(K, L) = bK^\alpha L^{1-\alpha}$, where $0 < \alpha < 1$, satisfies the equation

$$K \frac{\partial Q}{\partial K} + L \frac{\partial Q}{\partial L} = Q.$$

3. Compute all second-order partial derivatives.

a) $f(x, y) = \sin(x^2 + y^2)$; b) $f(x, y) = xe^{xy}$; c) $f(x, y) = x + \frac{x}{y}$;

d) $f(x, y) = y \ln(xy)$; e) $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$; f) $f(x, y, z) = \ln(x^2 + y^4 + z^6 + 1)$.

4. Let $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$. Compute $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$

5. Compute

a) $\frac{\partial^3 f}{\partial y \partial x^2}$ for $f(x, y) = \cos(xy)$; b) $\frac{\partial^5 f}{\partial x \partial y^2 \partial z^2}$ for $f(x, y, z) = e^{xy+z}$.

6. Let $f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$.

- a) Is f differentiable for $(x, y) \neq (0, 0)$?
 b) Find $f_x(0, 0)$ and $f_y(0, 0)$.
 c) Is f differentiable at $(0, 0)$?

7. Find the equation of the tangent plane to $z = \sqrt{17 - x^2 - y^2}$ at the point $(2, 3, 2)$.

8. Find the equation of the tangent plane to $z = \frac{8}{xy}$ at the point $(2, 1, 4)$.

9. Let $f(x, y) = (e^x - x) \cos y$. Find a vector which is perpendicular to the level curve of f through the point $(2, 3)$ in the direction in which f decreases most rapidly.

10. At what point on the surface $z = 1 + x^2 + y^2$ is its tangent plane parallel to the following planes?
 a) $z - 5 = 0$, b) $6x - 10y - z + 5 = 0$.

11. A differentiable function f has the property that $f(1, 3) = 7$ and $\text{grad } f(1, 3) = 2\vec{i} - 5\vec{j}$.

- a) Find the equation of the tangent line to the level curve of f through the point $(1, 3)$.
 b) Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(1, 3, 7)$.